

Demonstrating nonlocality induced teleportation through Majorana bound states in a semiconductor nanowire

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It was predicted by Tewari *et al* [Phys. Rev. Lett. **100**, 027001 (2008)] that a *teleportationlike* electron transfer phenomenon is one of the novel consequences of the existence of Majorana fermion, because of the inherently nonlocal nature. In this work we consider a concrete realization and measurement scheme for this interesting behavior, based on a setup consisting of a pair of quantum dots which are tunnel-coupled to a semiconductor nanowire and are jointly measured by two point-contact detectors. We analyze the teleportation dynamics in the presence of measurement backaction and discuss how the teleportation events can be identified from the current trajectories of strong response detectors.

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The search for Majorana fermions in solid states has been attracting a great deal of attention in the past years [1–8]. In solid states, it has been predicted that the Majorana bound states (MBSs) can appear for instance in the 5/2 fractional quantum Hall system [9] and the *p*-wave superconductor and superfluid [10]. In particular, an effective *p*-wave superconductor can be realized by a semiconductor nanowire with Rashba spin-orbit interaction and Zeeman splitting and in proximity to an *s*-wave superconductor [3–6]. This opens a new avenue of searching for Majorana fermions using the most conventional materials. Also, some demonstrating schemes were proposed based on various transport signatures, including the tunneling spectroscopy which may reveal characteristic zero-bias conductance peak [11, 12] and peculiar noise behaviors [13, 14], the nonlocality nature of the MBSs [15, 16], and the 4π periodic Majorana-Josephson currents [1–3, 17]. In the aspect of experiment, exotic signatures that may reveal the existence of MBSs have been observed in the system of semiconductor nanowire in proximity to an *s*-wave superconductor [18–21].

An inevitable consequence of the existence of Majorana zero modes is that the fermion quasiparticle excitations are inherently nonlocal. To be specific, let us consider a semiconductor nanowire in the topological regime which thus supports the MBSs at the two ends [3, 5, 6, 18], and denote the MBSs by Majorana operators γ_1 and γ_2 . They are related to the regular fermion operator in terms of $f^\dagger = (\gamma_1 + i\gamma_2)/\sqrt{2}$ and its Hermitian conjugate f . This connection implies some remarkable consequences. For instance, if an electron with energy smaller than the energy gap between the Majorana zero mode and other excited states is injected into the system, we can only have the excitation described by f and f^\dagger . This means that a single electron is “split” into two Majorana bound states which

are, however, spatially separated. In this work, instead of exploiting certain *indirect* transport signatures, we discuss a possible and very *direct* way to demonstrate this intrinsic *nonlocality* of the paired Majorana modes.

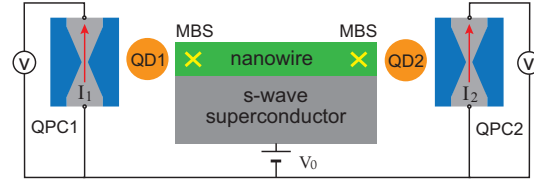


FIG. 1: Schematic setup of using two point-contact detectors to demonstrate the Majorana-nonlocality-induced *teleportationlike* electron transfer between two remote quantum dots. The semiconductor nanowire is in contact with an *s*-wave superconductor, so that under appropriate conditions a pair of Majorana bound states (MBS) are anticipated to appear at the ends of the nanowire. Here we show the schematic closed circuit, in which the chemical potential of the superconductor and the bias voltages across the detectors are explicitly defined.

The proposed scheme is schematically displayed in Fig. 1, where the two MBSs, generated at the ends of the nanowire, are tunnel-coupled to two quantum dots (QDs), respectively. Moreover, the QDs are jointly probed by the nearby quantum-point-contact (QPC) detectors. This proposal is motivated by the nowadays state-of-the-art technique, which enables the QPC current to sensitively probe an extra single electron in the nearby quantum dot [22]. In Ref. [15], an equivalent “dot-MBSs-dot” system is analyzed by assuming an extra electron initially in one of the QDS and considering its transmission through the MBSs in a vanished hybridization limit. Corresponding to the nanowire realization in Fig. 1, their prediction indicates that, in a “long-wire” limit, the electron can transmit through the nanowire on a finite (short) timescale, revealing thus a “teleportation” or “superluminal” phenomenon. In our present work, following Ref. [15], we call this *ul-*

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trafast transfer behavior *teleportation*, which is actually a remarkable consequence of Majorana's nonlocality. Related to the scheme of joint-measurements shown in Fig. 1, we will carry out the teleportation dynamics under the influence of measurement backaction, and discuss how the teleportation events can be identified from the current trajectories of strong response detectors. Also, we will present an interpretation/understanding to the “teleportation” (“superluminal”) issue.

Model.— The setup of Fig. 1 can be described by the following Hamiltonian

$$H = H_{sys} + H_{pc}. \quad (1)$$

The *system* Hamiltonian, H_{sys} , describes the MBSs plus the single-level QDs and their tunnel coupling as follows [2, 12–15]

$$H_{sys} = i \frac{\epsilon_M}{2} \gamma_1 \gamma_2 + \sum_{j=1,2} [\epsilon_j d_j^\dagger d_j + \lambda_j (d_j^\dagger - d_j) \gamma_j]. \quad (2)$$

Here γ_1 and γ_2 are the Majorana operators associated with the two MBSs at the ends of the nanowire. The two MBSs interact with each other by a strength $\epsilon_M \sim e^{-L/\xi}$, which damps exponentially with the length (L) of the nanowire, with a characteristic length of the superconducting coherent length (ξ). $d_1(d_1^\dagger)$ and $d_2(d_2^\dagger)$ are the annihilation (creation) operators of the two single-level quantum dots, while λ_1 and λ_2 are their coupling amplitudes to the MBSs. In practice, it will be convenient to switch from the Majorana representation to the regular fermion one, through the transformation of $\gamma_1 = i(f - f^\dagger)$ and $\gamma_2 = f + f^\dagger$. We can easily check that f and f^\dagger satisfy the anti-commutative relation, $\{f, f^\dagger\} = 1$. After an additional local gauge transformation, $d_1 \rightarrow i d_1$, we reexpress Eq. (2) as

$$H_{sys} = \epsilon_M (f^\dagger f - \frac{1}{2}) + \sum_{j=1,2} [\epsilon_j d_j^\dagger d_j + \lambda_j (d_j^\dagger f + f^\dagger d_j) - \lambda_1 (d_1^\dagger f^\dagger + f d_1) + \lambda_2 (d_2^\dagger f^\dagger + f d_2)]. \quad (3)$$

It should be noticed that the tunneling terms in this Hamiltonian only conserve charge modulo $2e$. This reflects the fact that a pair of electrons can be extracted out from the superconductor condensate and can be absorbed by the condensate.

The other Hamiltonian in Eq. (1), H_{pc} , is for the two point-contacts which reads

$$H_{pc} = \sum_{j=1,2} \sum_{l_j, r_j} [(\epsilon_{l_j} c_{l_j}^\dagger c_{l_j} + \epsilon_{r_j} c_{r_j}^\dagger c_{r_j}) + (w_j c_{l_j}^\dagger c_{r_j} + \text{H.c.})]. \quad (4)$$

This Hamiltonian simply describes electron tunneling through a potential barrier between two electronic reservoirs (with electron creation and annihilation operators, $c_{l_j(r_j)}^\dagger$ and $c_{l_j(r_j)}$). We assume that the tunneling amplitudes (w_j) are approximately of energy independence. Thus w_j does not depend on the associated states “ l_j ” and

“ r_j ”. However, in w_j we should include the effect of the nearby quantum dot, since its occupation would change the tunneling amplitudes. We account for this effect in terms of $w_j = \Omega_j + \Delta \Omega_j d_j^\dagger d_j$.

Teleportation.— Let us consider the transfer problem of an *extra* electron between the two quantum dots, which is assumed initially in the left quantum dot.

In this part we assume a simpler setup in the absence of the point-contact detectors [15].

In particular, we consider the weak interaction limit $\epsilon_M \rightarrow 0$, in order to reveal the remarkable *teleportation* behavior. Using the transformed representation, $|n_1, n_M, n_2\rangle$ describes the possible charge configuration of the dot-MBSs-dot system, where $n_{1(2)}$ and n_M denote, respectively, the electron number (“0” or “1”) in the left (right) dot and the central MBSs. Totally, we have eight basis states, which can be divided into two subspaces: $|100\rangle, |010\rangle, |001\rangle, |111\rangle$ with odd parity (electron numbers); and $|110\rangle, |101\rangle, |011\rangle, |000\rangle$ with even parity. Associated with our specific initial condition, we will only have the odd-parity states involved in the state evolution. Moreover, for simplicity, we assume $\lambda_1 = \lambda_2 = \lambda$ and $\epsilon_1 = \epsilon_2 = 0$ throughout this work.

Simple calculation can give the occupation probabilities of the left and right dots, respectively, as $P_1(t) = \cos^2(\lambda t)$ and $P_2(t) = \sin^2(\lambda t)$. Here, for each of the probabilities, it contains two possible occupations: $|100\rangle$ and $|111\rangle$ for $P_1(t)$; $|001\rangle$ and $|111\rangle$ for $P_2(t)$. Now, we introduce (extract) the partial probability $P_2^{(1)}(t) = |\langle 001 | e^{-iH_{sys}t} | 100 \rangle|^2$ from $P_2(t)$, which has also a simple form, $P_2^{(1)}(t) = \sin^4(\lambda t)$. Similarly, we may define $P_2^{(2)}(t) = |\langle 111 | e^{-iH_{sys}t} | 100 \rangle|^2$, which can be obtained simply by $P_2^{(2)}(t) = P_2(t) - P_2^{(1)}(t) = \sin^2(\lambda t) \cos^2(\lambda t)$. Based on these simple manipulations, of great interest is the result of $P_2^{(1)}(t)$, since it implies that, even in the limit of $\epsilon_M \rightarrow 0$ (very “long” nanowire), the electron in the left dot can transmit through the MBSs and appear in the right dot on some finite (short) timescale. This is the remarkable “teleportation” phenomenon discussed in Ref. [15] which, surprisingly, holds a “superluminal” feature. In the following, to prove this teleportation behavior, we propose to use QPC detectors to perform a *coincident* measurement of both the occupation numbers of the left and the right dots. This type of measurement can distinguish the process responsible for $P_2^{(1)}(t)$ from that responsible for $P_2^{(2)}(t)$.

Demonstration.— Now we turn to the measurement setup of Fig. 1. Physically, the measurements will cause backaction on the charge transfer dynamics in the central dot-MBSs-dot system. This effect can be described by a master equation, formally expressed as [23]

$$\dot{\rho} = -i\mathcal{L}\rho - \mathcal{R}\rho. \quad (5)$$

The first term denotes $\mathcal{L}\rho = [H_{sys}, \rho]$, and the second term describes the measurement backaction. More specifically, $\mathcal{R}\rho = \frac{1}{2} \sum_{j=1,2} \{ [w_j^\dagger, \tilde{w}_j^{(-)} \rho - \rho \tilde{w}_j^{(+)}] + \text{H.c.} \}$, where $\tilde{w}_j^{(\pm)} = C_j^{(\pm)} (\pm \mathcal{L}) w_j$. $C_j^{(\pm)} (\pm \mathcal{L})$ are the

Liouvillian counterparts of the QPC spectral functions $C_j^{(\pm)}(\pm\omega)$, which were obtained explicitly in Ref. [23]. In this work, we restrict to a wideband limit and large bias condition for the point-contact detectors, which allow us to approximate $C_j^{(\pm)}(\pm\omega)$ by $C_j^{(\pm)}(0)$. More explicitly, we have [23]: $C_j^{(\pm)}(0) = \pm 2\pi g_L g_R e V_j / (1 - e^{\mp\beta e V_j})$, where $g_{L(R)}$ is the density-of-states of the QPC reservoir, V_j is the applied voltage, and β is the inverse temperature. Under these considerations, Eq. (5) becomes the Lindblad-type master equation, with $\mathcal{R}\rho = -\sum_{j=1,2} \Gamma_j \mathcal{D}[n_j]\rho$. Here, $n_j = d_j^\dagger d_j$, $\mathcal{D}[n_j]\rho = n_j \rho n_j - \frac{1}{2}\{n_j n_j, \rho\}$, and the backaction-induced dephasing rate

$$\Gamma_j = 2\pi g_L g_R |\Delta\Omega_j|^2 e V_j \coth(\beta e V_j / 2). \quad (6)$$

At zero temperature and introducing the tunneling coefficients $\mathcal{T}_j = 4\pi^2 g_L g_R |\Omega_j|^2$ and $\mathcal{T}'_j = 4\pi^2 g_L g_R |\Omega'_j|^2$ (here we denote $\Omega'_j = \Omega_j + \Delta\Omega_j$), we can reexpress the dephasing rate as $\Gamma_j = (\sqrt{\mathcal{T}_j} - \sqrt{\mathcal{T}'_j})^2 e V_j / 2\pi$. In the latter feasibility estimates, we will use this compact expression.

To reveal the teleportation behavior, we investigate the steady-state correlation function

$$S(t) = \langle M_2(t) M_1(0) \rangle_{ss}, \quad (7)$$

where the measurement operators are designed as $M_1 = n_1(1 - n_2)$ and $M_2 = n_2(1 - n_1)$. The meaning of $S(t)$ is clear. In steady state, at some chosen *initial* moment ($t = 0$), if we find the left (right) dot occupied (unoccupied), $S(t)$ predicts the probability of finding the reversed occupation at time t , say, the left dot empty and the right dot occupied. This simply indicates an electron transfer (“teleportation”) between the *distant* dots, separated by the *long* nanowire that supports the MBSs in the limit $\epsilon_M \rightarrow 0$.

Applying the master equation approach, we can easily calculate the steady-state correlator $S(t)$. Restricted in the odd-parity subspace, one may first solve the master equation to obtain the steady state ρ_{ss} . Then, starting with ρ_{ss} and after the “ M_1 ” measurement, we propagate the resultant “initial” state $|100\rangle\langle 100|$. At the moment t , the probability of finding the state $|001\rangle\langle 001|$ is right the $S(t)$, which is given by $S(t) = \text{Tr}[M_2 \rho(t)]$. In Fig. 2 we plot the result of $S(t)$ in both the time domain and frequency space. The damping oscillations in Fig. 2(a) show the teleportation dynamics between the quantum dots, under the influence of measurement backaction. We remind again that this result is obtained in the limit of $\epsilon_M \rightarrow 0$ which corresponds to a *long* nanowire. So any change of $S(t)$ on short time scales indicates a *teleportationlike* behavior. Notice also that the maximal height of the peaks, less than unity, is limited by the probability of finding “ $|100\rangle\langle 100|$ ” in the steady state ρ_{ss} . In Fig. 2(b) the same result in frequency space, i.e., the Fourier transform of $S(t)$, is shown. We notice a prominent “dip” at $\omega = 2\lambda$ and a relatively small “peak” at $\omega = 4\lambda$, which are originated, respectively, from the two harmonics involved in $S(t)$. A simple

way to understand this is from the Fourier spectrum of $P_2^{(1)}(t) = \sin^4(\lambda t)$, which contains two harmonics with the above frequencies.

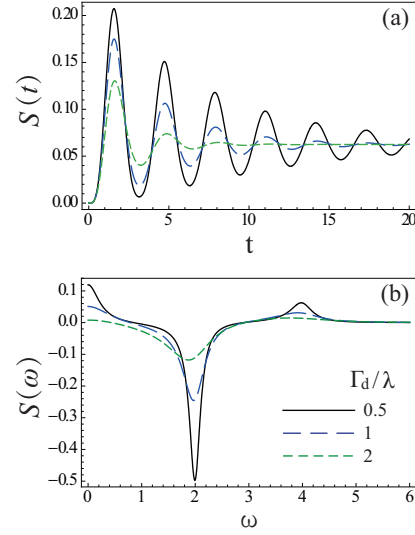


FIG. 2: Stationary cross-correlation in (a) the time domain and (b) the frequency space. We assume a symmetric setup and denote the dephasing rate of Eq. (6) by Γ_d . The units of frequency and time are, respectively, λ and λ^{-1} .

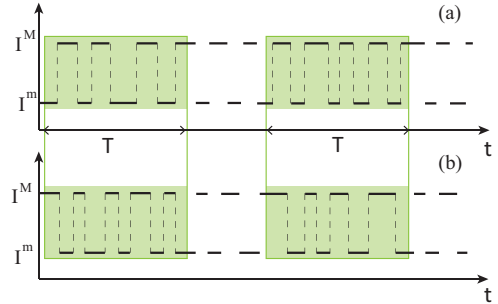


FIG. 3: Schematic output-currents in a strong-response regime. Shaded are two examples of electron transfer from the state $|100\rangle$ to $|001\rangle$. We consider a symmetric setup which allows a common maximum (minimum) current of I^M (I^m).

The above dot-occupation correlator, $S(t)$, is closely related to the following cross correlation of detector currents, $S_I(t) = \langle M_{I,2}(t) M_{I,1}(0) \rangle_{ss}$, where $M_{I,1} = (I_1^M - I_1)(I_2 - I_2^m)$ and $M_{I,2} = (I_1 - I_1^m)(I_2^M - I_2)$. I_j^M is the (maximum) current through the j th point-contact when the nearby dot is empty, while I_j^m is the (minimum) current when the dot is occupied. Based on this type of correlation-function measurement, in a relatively weak or intermediate response regime, i.e., the detector does not *clearly* distinguish the occupation/non-occupation of the nearby quantum dot, one can obtain the result as shown in Fig. 2. This corresponds to the continuous weak measurement, which was also extensively studied in the measurement of solid-state qubit [23, 24]. In particular, the present two-detector coincident measurement can be regarded as

a counterpart of the cross-correlation measurement of a solid-state qubit by using two QPC detectors [25].

On the other hand, in a strong-response regime, the detector can definitely distinguish whether the nearby quantum dot is occupied or not. In this case, the output currents appear as the telegraphic signals, as schematically shown by Fig. 3, where we assumed a symmetric setup so that the maximum (minimum) current is commonly denoted by I^M (I^m). Holding this type of data record, one has actually *unraveled* the ensemble-averaged result. In doing this, we should pay particular attention to the process from $|100\rangle$ to $|001\rangle$, passing through an intermediate state $|010\rangle$ in a short time interval T . Or, more strikingly, as $T \rightarrow 0$ there may exist a sudden “jump” from $|100\rangle$ to $|001\rangle$. These events, very clearly, reveal the teleportation phenomenon.

The above teleportation is an *unusual* consequence of the quantum nonlocality of Majorana fermion. Moreover, the *teleportation* process is seemingly indicating a *superluminal* phenomenon. In Ref. [15], in order to rule out such possibility, it was argued that, since a *classical exchange of information* (the result of the coincident measurement) is necessary, there is no superluminal transfer of information in the observation of the teleportation effect. However, we may notice here that, in the above joint measurements, there is no need to perform the classical exchange of information to confirm the result being $|001\rangle$ but not $|111\rangle$. That is, we first keep the data record of the joint measurements (as shown in Fig. 3). Then, we check whether there exists such process that switches directly from $|100\rangle$ to $|001\rangle$, but not through an intermediate state $|111\rangle$. Since through our above analysis we do expect to find such result (event) from the data record, and obviously that event is *objective* despite that we did not confirm it at *that moment* via any classical communication, now the interesting problem is how we should interpret this result? Unlike the argument in Ref. [15], we would like to provide a different understanding. Since the Majorana fermion is a quasiparticle excitation in the presence of other electrons (background condensate of electrons), we cannot conclude that the electron appeared in the right quantum dot is the one initially in the left dot. This excludes the possibility of superluminal electron transfer be-

tween the two remote quantum dots. Therefore, the above result only demonstrates the Majorana’s nonlocality nature, but does not imply a superluminal phenomenon.

Feasibility.— The semiconductor InSb nanowire that may support the MBSs has been utilized in the recent experiments [18]. The InSb nanowire has a large g -factor (with $g \simeq 50$), and a strong Rashba-type spin-orbit interaction (with energy $\sim 50 \mu\text{eV}$). Under proper magnetic field (e.g., 0.15 Tesla), the Zeeman splitting starts to exceed the *induced* superconducting gap $\Delta \simeq 200 \mu\text{eV}$ and thus to support the emergence of MBSs at the ends of the nanowire. Also, a low temperature such as $T = 100 \text{ mK}$ can suppress the thermal excitation of the Majorana zero mode to higher energy states. If we tune the dot-MBS coupling (λ) to, for instance, $20 \mu\text{eV}$, the following estimates show that the coherent oscillations of charge transfer between the remote dots can be observed in the proposed setup. Based on Eq. (6) and assuming a symmetric setup, the measurement-backaction-induced dephasing rate reads $\Gamma_d = (\sqrt{\mathcal{T}} - \sqrt{\mathcal{T}'})^2 V_d / 2\pi$, and the measurement “signal” is simply given by $\Delta I = I^M - I^m = (\mathcal{T} - \mathcal{T}') V_d / 2\pi$. Here \mathcal{T} (\mathcal{T}') denotes the transmission coefficient through the point-contact barrier, corresponding to the nearby dot being empty (occupied). If we assume $\mathcal{T} = 0.16$, $\mathcal{T}' = 0.09$ and $V_d = 1 \text{ mV}$, a simple estimate gives $\Gamma_d \simeq 10 \mu\text{eV}$ and $\Delta I \simeq 2.4 \text{ nA}$. These results, reasonably, favor an implementation of the proposed measurement scheme.

In summary, we analyzed a concrete realization and measurement scheme to demonstrate the *teleportation-like* electron transfer phenomenon mediated by Majorana fermion, as a remarkable consequence of its inherently nonlocal nature. Since all the major aspects of the proposed scheme are seemingly within the reach of the state-of-the-art experiments in nowadays laboratories, we expect that this striking phenomenon can be demonstrated experimentally in the forthcoming future.

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- [1] A. Y. Kitaev, Physics-Uspekhi **44**, 131 (2001).
 - [2] R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Phys. Rev. Lett. **105**, 077001 (2010).
 - [3] Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. **105**, 177002 (2010).
 - [4] C. Zhang, S. Tewari, R. M. Lutchyn, and S. Das Sarma, Phys. Rev. Lett. **101**, 160401 (2008).
 - [5] J. D. Sau, R. M. Lutchyn, S. Tewari, and S. Das Sarma, Phys. Rev. Lett. **104**, 040502 (2010).
 - [6] J. D. Sau, S. Tewari, and S. Das Sarma, Phys. Rev. B **85**, 064512 (2012).
 - [7] L. Fu and C. L. Kane, Phys. Rev. Lett. **100**, 096407 (2008).
 - [8] J. Alicea, Phys. Rev. B **81**, 125318 (2010).
 - [9] G. Moore and N. Read, Nuclear Physics B **360**, 362(1991).
 - [10] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
 - [11] K. Sengupta, I. Zutic, H. J. Kwon, V. M. Yakovenko, and S. Das Sarma, Phys. Rev. B **63**, 144531 (2001).
 - [12] D. E. Liu and H. U. Baranger, Phys. Rev. B **84**, 201308 (2011).
 - [13] C.J. Bolech and E. Demler, Phys. Rev. Lett. **98**, 237002 (2007).
 - [14] K. T. Law, P. A. Lee, and T. K. Ng, Phys. Rev. Lett. **103**, 237001 (2009).
 - [15] S. Tewari, C. Zhang, S. Das Sarma, C. Nayak, and D. H. Lee, Phys. Rev. Lett. **100**, 027001 (2008).
 - [16] J. Nilsson, A. R. Akhmerov, and C. W. J. Beenakker, Phys.

- Rev. Lett. **101**,120403 (2008).
- [17] L. Fu and C. L. Kane, Phys. Rev. B **79**, 161408 (2009).
 - [18] V. Mourik *et. al*, Science **336**, 1003 (2012).
 - [19] M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, arXiv:1204.4130.
 - [20] L. P. Rokhinson, X. Liu, and J. K. Furdyna, arXiv:1204.4212.
 - [21] A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, arXiv:1205.7073.
 - [22] S. Gustavsson *et. al*, Phys. Rev. Lett. **96**, 076605 (2006); T. Fujisawa *et. al*, Science **312**, 1634 (2006).
 - [23] X. Q. Li, W. K. Zhang, P. Cui, J. S. Shao, Z. S. Ma, and Y. J. Yan, Phys. Rev. B **69**, 085315 (2004); X. Q. Li, P. Cui, and Y. J. Yan, Phys. Rev. Lett. **94**, 066803 (2005).
 - [24] Yu. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
 - [25] A. N. Jordan and M. Büttiker, Phys. Rev. Lett. **95**, 220401 (2005).